SACE Mathematical Methods External Examination 2018

Before we start – some general marking guidelines that we employ when trying to ensure fair and consistent marking.

My recommendation is that much of this advice on marking principles (but not all) should apply to <u>internal</u> SAT assessment also, which implies that a mark scheme should be generated (rather than merely the solutions from which to mark) so different teachers mark common assessment equitably, and specific skills are rewarded rather than marks deducted. Assessment should allow students to properly showcase their knowledge, skills and understanding of the course.

- The marks scheme indicates the knowledge, skills and/or understanding (KSU) required to earn each mark in each paper. The rationale of the original question should be with the goal that the candidate might demonstrate these KSU.

- "Has the student, in their solution, provided evidence that they possess the knowledge, skills or understanding for which this mark is allocated?"

- The goal is to reward relevant work and skills, fairly and equitably for all candidates.

- The "Burden of Proof" is on the student to show evidence of KSU.

- The first trivial error that a student makes per question should not be penalised, however subsequent trivial errors should result in a loss of marks .

- Work that “follows through” from an error is still able to demonstrate KSU and be rewarded accordingly. Follow through that leads to results that are clearly incorrect or infeasible cannot be awarded the mark for that result unless accompanied by a student comment of the inappropriateness of their solution. Follow through leading to a contradiction with a given result (such as might occur in a ‘show that’ scenario) cannot be considered to be trivial errors.

- Multiple solutions are marked and the best mark awarded except in worded responses, but a one mark penalty will be imposed should a “complete and incorrect” solution be provided alongside a “complete and correct” solution with no indication that the student knows which is correct, since they have shown all the KSU but the ability to differentiate between correct and incorrect mathematics.

- Worded responses that contain evidence of KSU, and also counter evidence of the same KSU (where a candidate is directly contradicting themselves), have not established the KSU needed for the mark.

- The degree to which a “Final Answer Only” (FAO) provides sufficient evidence of the KSU varies from one question to another. This largely determined by whether evidence of required procedural skills can be inferred from their correct answer.

- Students who provide results to less than 3 significant figures should not be penalised unless the lack of accuracy implies that they have not provided evidence of the KSU, with the caveat that there will be an identified examination question where a required accuracy will be enforced.

It is worth noting that the split of the 3 hour examination into two separate papers allows the SACE board to gain more consistency in the marking process, since individual markers are responsible for less questions.

Some general comments from the exam

The standard deviation of the percentage of available marks for a question (%SD) on all but question 2 was more than 20%. A significant spread was evident on each question, not just on the more challenging problems.

Examination markers aim to award marks for evidence of student understanding in responding to examination questions wherever possible, however, students should be advised not to cross out their responses or attempted responses to questions in the examination booklet, unless they are confident that no part of their response should be considered by the marker.

If a student crosses out a response and then decides that it was the correct (or the most correct) answer, then the student should indicate clearly to the marker which part of their response should be considered. This could be done by circling or highlighting the response, or part of the response that the student want to be considered and write “please mark this work”. Students do not need to rewrite their answers in this case, unless the crossing out has rendered the response unreadable.

Question 1

This question was a successful start to the examination for the vast majority of students, with 47% earning full marks and nearly 95% earning half marks or better.

The more success responses:

- featured the correct use of the Chain, Product and Quotient Rules
- used of the laws of logarithms as a means for simplifying the differentiation process.

The less successful responses:

- made errors with indices and/or signs in part (a)
- did not use the Product Rule when differentiating $\ln(u \times v)$.

Question 2

This question was the most accessible question in the paper, with over 80% of students earning full marks.

The more successful responses:

- made the correct selection of graphs in each part

The less successful responses:

- erroneously selected the graph representing the complement of the probability given, perhaps due to misunderstanding the inequality sign as it is used to represent probability.
- unnecessarily included written justifications for their answers, wasting valuable time.

Question 3

Overall, this question was an accessible one, utilising a relatively simple trigonometric function in a familiar context and including opportunities to utilise graphics technology.

This led to nearly three quarters of students earning better than half marks.

The differentiation of a trigonometric function and the need to use detailed, bi-directional language when providing an interpretation for such a function provided an opportunity for differentiation between students, with only 20% earning full marks.

The more successful responses:

- correctly used the Chain Rule to differentiate the trigonometric function
- used graphics technology to help them graph correctly using the correct view window
- provided a sufficient degree of detail in part (a)(ii) .

The less successful responses:

- did not use contextual language in part (a)(ii)
- used uni-directional language in part (a)(ii) i.e. talked about “the flow into the tank” rather than “into and out of the tank” or equivalent.
- focussed on the maximum when answering part (c) rather than the point of inflection.

Part c) (i) FAO 2

Question 4

This question contained some routine statistical computations presented in a straightforward context, but also asked students to assess claims using these computations as well as to describe, in some detail, the central limit theorem in action.

As a result, less than a quarter of students earned half marks or below, and only a third of students earned 7 or 8 out of 8.

The more successful responses:

- explicitly established the connection between a sample sum of 450 g and a sample mean of 18 g
- provided a detailed enough response in part (e), including the fact that an increased sample size leads to an increasingly normal distribution of sample means.

The less successful responses:

- did not identify that this question was dealing with sample means rather than sample sums.
- did not realise that Histogram A was for sample size 15 but from part (c) onwards the sample size was 25.

Question 5

This first principles question focussed on a familiar, but not simple function that was attempted well by students. Those well versed in this aspect of the curriculum did well, with over half of all students earning full marks and only 30% earning 3 marks or fewer.

The more successful responses:

- used the limit notation in line with the expectations of this procedure
- communicated all the required steps, making sure that their procedure was clear.

The less successful responses:

- were unable to make a common denominator as required by this form of question
- made errors when expanding $-5(x+h)^2$, failing to multiply through by the 5 and/or the negative.

Question 6

The first appearance in the paper of a parameter marked a step up in difficulty.

This question proved difficult to those students not confident with working in terms of a parameter, with 20% of students earning 0 or 1 mark. Those better prepared to handle such a parameter found the question accessible, with a third of students earning full marks.

The more successful responses:

- were comfortable with the use of t instead of x as the explanatory variable
- could distinguish between variables and constants.

The less successful responses:

- did not solve the equation for t in part (a)
- mixed up horizontal and vertical intercepts
- demonstrated some confusion with the use of t instead of x as the explanatory variable.

Part b) (i) FAO 1

Question 7

At this point in the paper, the questions became less routine and familiar in presentation and content.

This increase in difficulty meant that nearly 30% of students did not make significant headway, earning 2 or fewer marks out of 8, perhaps not confident with the formal algebraic structures.

Those who made their way into the question, having at least partial success with the proof and earning 5 or more marks, consisted of 43% of students, with nearly 30% of students earning full marks

The more successful responses:

- used the Chain Rule, rather than using the Quotient Rule, to answer part (a)
- did not make up a Quotient Rule for integration in part (b)(iii) but recognised that the earlier derivative offered a way to tackle the integral required.

The less successful responses:

- failed to see the connection between part (a) and (b)(iii)
- struggled to correctly differentiate in part (a).

Part a) FAO 2

Question 8

This question contained routine statistical computations, the interpretation of a confidence interval and the non-routine use of a confidence interval's end points as mean estimates.

Collectively, this meant that 80% of students earned half marks or better but only 19% earned 7 or 8 out of 8, due to the more challenging nature of part (d).

The more successful responses:

- calculated probabilities and confidence intervals efficiently
- used statistical notation correctly
- discussed the relationship between the 'old mean' and the confidence interval in terms of 'lower' rather than 'outside'.

The less successful responses:

- were confused as to which 'mean' value to compare to the confidence interval
- did not see the confidence interval endpoints as mean estimates in part (d)

Part a) FAO 2

Question 9

In spite of the inverse thinking required to comprehend the graphical relationship between $f'(x)$ and $f(x)$, students were able to access this thinking in a way that suggests that this type of question was less foreign to students in 2018 compared to previous years.

60% earned 4 or more out of 7 and 36% earned 6 or more.

The more successful responses

- ascertained that the graph provided was of $y = f'(x)$ and answered part (a) accordingly.

The less successful responses

- did not draw the graph through the origin as instructed
- did not extend the graph beyond $x = 0$ and $x = b$ to show the turning point nature in these locations
- added extra stationary and inflection points.

Question 10

This question was a relatively approachable start to Part 2 of the examination, covering some routine, but not trivial, course content – mean and standard deviation of a continuous random variable, differentiation and integration and trigonometric functions.

Overall, students showed a good grasp of this content, with two thirds of students earning more than half marks and 20% earning full marks

The more successful responses:

- used the Product Rule successfully in part (a)
- identified and used the connection between part (a) and part (c)(ii).

The less successful responses:

- did not use notation to correctly indicate when the process of differentiation or integration had taken place i.e. did not switch clearly from $g(x)$ to $g'(x)$ or did not discontinue the use of the integral symbol at the appropriate juncture
- made errors with the formulae for mean and standard deviation.

Part d) FAO 2

Question 11

This question provided access via familiar algorithms, like the calculation of the area under the curve using rectangles, but also asked students to work with exponential equations using logarithms and to reflect on graphical information in a non-routine fashion.

As such, students had a chance to demonstrate their understanding of course content at a range of levels. Pleasingly, 70% of students earned half marks or better and nearly a quarter earned 9 or 10 out of 10.

The more successful responses:

- were able to work with exponential equations, using log laws as required
- demonstrated understanding of the concepts and language of convexity and concavity.

The less successful responses:

- read function values off the graph
- assumed that the over and underestimates would be equidistant from the exact area
- made computational errors when calculating the underestimate.

Part a) (ii) FAO 2

Part b) (i) FAO 2

Question 12

This question focussed on the use of the laws of logarithms, and the resulting relationship between two functions, a relationship that is best understood as a vertical dilation or 'stretch'.

The laws of logarithms, and their use, has often been an area that students have had difficulty with, however, students need to be familiar with the ideas and language of transformations due to their presence in sub-topic 4.2 of the subject outline.

This question proved to be the most challenging one in the examination for students. Over a third of students earned 0 or 1 mark for this question and only 16% earned 5 or 6 out of 6

The more successful responses:

- described the relationship between the functions using the idea of vertical 'stretch', using formal or informal language of transformations in part (b)(ii).

The less successful responses:

- added additional brackets to the function that they graphed, effectively squaring the logarithm term, rather than finding the logarithm of the squared term. This gave rise to a 'tick' shaped graph in part (b).
- were not able to use the laws of logarithms effectively in parts (a) and/or (c).
- either described the relationship between the functions ($y_2 = 3y_1$) or described specific features that the graphs shared (same x intercept etc) rather than the relationship between the graphs in their entirety.

Part c) FAO 0

Question 13

This question thoroughly examined the tangent to a trigonometric curve, expecting students to work in terms of exact values.

Some students struggled to make much headway with this question, with 38% of students earning 2 or fewer marks out of 11. Over 40% earned better than half marks, and 14% earned full marks.

The more successful responses:

- appreciated that, when working with trigonometric functions, there is often more than one solution and, in this case, the 'second solution' was required.
- worked successfully with radian values in terms of π as required.

The less successful responses:

- used decimal approximations
- provided limited or no reasoning in part (b), despite it being worth 4 marks. In such cases, a solution heavily reliant on electronic technology needs proper documentation i.e. the equation being solved, the list of solutions obtained by technology and the reasons for the selection of required solutions.

Question 14

This question provided an accessible introduction into a thorough examination of binomial probabilities and their ability to help us draw informed conclusions to statistical based claims made in context.

Due to the routine elements of the question, 82% of students earned at least 4 out of 14 marks. The interpretations and more complex process required later in the question meant that only 35% of students earned half marks or better.

The more successful responses:

- used clear mathematical notation to communicate their methods, including identifying the distribution being used.
- were able to make the required interpretations in parts (e) and (g).

The less successful responses:

- made errors when computing using the complement to perform the calculation in part (b)(ii)
- were unable to execute the trial and error computations required in part (d)
- made comments about the computed probability, but did not comment upon Ari's claim as they were asked to.

Part b) (ii) FAO 2

Part f) FAO 2 if 5 sig figs shown

Question 15

The routine calculations required throughout this question meant that most students achieved a good degree of success, with nearly three quarters earning 6 or more marks out of 14. The non-routine probability calculations in part (b), requiring the use and understanding of combinations that underpin the binomial distribution, meant that only 16% of students earned 12 or more marks.

The more successful responses:

- were able to perform the calculations in parts (a), (c), (d) and (e) accurately
- were able to tackle the non-routine probability calculations in part (b) using combinations.

The less successful responses:

- tried to use incorrect distributions, or incomplete lists, in part (b)(ii).

Part a) (i) & (ii) both FAO 2 – but don't expect this in future!

Part b) (i) FAO 2

Part e) FAO 0

Question 16

Some students struggled to make headway in this question, with nearly 20% earning 0 or 1 mark, perhaps reflecting the question's position in the paper. 46% of students persevered, earning better than half marks, but only 17% earned 10 or more out of 12, reflecting the difficulty of the later parts of the question.

The more successful responses:

- made largely successful attempts at obtaining the required result in part (c)
- answered part (a) to the required degree of accuracy.

The less successful responses:

- correctly, but inefficiently, answered part (a) by finding the first and second derivative
- surmised the answer to part (d) without using the equation in part (c) as required.

Part e) FAO 0